

Design of Tubular Flow Reactors for Monodisperse Aerosol Production

Design equations are derived for calculating the properties of an aerosol generated in a tubular flow reactor operated at a constant rate of formation of condensable monomer. These equations lead to a set of characteristic dimensionless groups. Independently, it is shown that nearly monodisperse aerosols can be generated by: (1) separating particle formation and growth processes, (2) operating with a narrow residence time distribution, and (3) minimizing monomer concentration gradients. These criteria can be used in solving the design equations over ranges of the dimensionless groups suitable for monodisperse aerosol production. In general, high rates of monomer formation are desirable, subject to the desired particle size and concentration, but an upper bound may result from coagulation.

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Introduction

Narrow particle size distributions are desirable for pigments (Herrmann, 1976), carbon black (Medallia and Riven, 1976; Dannenberg, 1971), and ceramic powders used for the production of ceramic parts (Kingery, 1960; Bowen, 1980). In the laboratory, monodisperse particles are required for instrument calibration and for fundamental studies of aerosol behavior.

Previous theoretical studies have addressed the general problem of controlled production of aerosol particles (Pratsinis et al., 1986a; Kotas et al., 1986). The goal of the present study was to develop a procedure for designing tubular flow reactors for the production of high concentrations of monodisperse particles in the size range 10 \AA to $10 \text{ }\mu\text{m}$. Moments of the particle size distribution were used to describe aerosol production. Particle formation, particle growth from the free molecule to the continuum regime, convection, and radial diffusion of the condensing species were included. The use of batch reactor theories (tubular flow theory with a flat velocity profile and no monomer diffusion) as a simplified method for the design of tubular flow reactors was investigated.

Basic Model

The performance of tubular aerosol flow reactors can be described in terms of moments of the particle size distribution

Previous calculations were based on a particle growth law valid for $d_p \ll 0.065 \text{ }\mu\text{m}$ for typical conditions (Friedlander, 1983; Kotas et al., 1986; and Pratsinis et al., 1986a). However, many applications of aerosol reactors involve production of particles with average diameters of 0.1 to $1.0 \text{ }\mu\text{m}$ or greater. For example, particle diameters of approximately $0.5 \text{ }\mu\text{m}$ are desirable for pigments (Herrmann, 1976) and for powders used for production of ceramic parts (Isakoff, 1984; Sanders, 1984). Thus, we have extended previous calculations to describe particle growth in the transition and continuum regimes.

The basic processes are as follows. A condensable species (monomer) is formed by chemical reaction in a tubular flow reactor. The monomer concentration increases as the gas flows through the reactor. Eventually, a sufficient supersaturation is reached to produce particles by homogeneous nucleation. Particle growth and molecular diffusion to the walls consume monomer, thereby lowering the saturation ratio and halting new particle formation. After this point, only particle growth and diffusion of the monomer to the walls occur. The analysis is based on the same assumptions as the analysis of Pratsinis et al. (1986a) except that in this work particle growth is described by the continuum regime growth law with the Fuchs-Sutugin correction factor.

Monomer balance

The monomer concentration in the reactor changes as the result of monomer formation at rate R' , loss to particle formation at rate $I'k^*$, loss to condensational particle growth at rate

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R'_c , radial diffusion, and convection. The monomer balance can be written as

$$2\bar{u}f(r)\frac{\partial n_1}{\partial x} = D_1\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial n_1}{\partial r'}\right) + R' - I'k^* - R'_c \quad (1)$$

where $f(r) = 1/2$ for a flat velocity profile and $f(r) = 1 - r^2$ for a parabolic velocity profile.

The rate $R'_{c,s}$ at which a single particle of size d_p consumes vapor molecules is given by Friedlander (1977):

$$R'_{c,s} = 2\pi D_1 d_p (n_1 - n_s S^{d_p^*/d_p}) F(Kn) \quad (2)$$

The Fuchs-Sutugin correction factor $F(Kn)$,

$$F(Kn) = \frac{1 + Kn}{1 + 1.71Kn + 4/3Kn^2} \quad (3)$$

is used to account for noncontinuum effects. The Knudsen number Kn is defined as

$$Kn = \frac{6D_1}{\bar{c}_1 d_p} \quad (4)$$

Hence Eq. 2 approaches the monomer consumption rate obtained from kinetic theory for $Kn \rightarrow \infty$ and the monomer consumption rate obtained using the continuum regime growth law for $Kn \rightarrow 0$. The term $S^{d_p^*/d_p}$ accounts for the Kelvin effect.

The total rate of monomer consumption by particle growth, R'_c , is obtained by integrating the monomer consumption rate for a single particle, $R'_{c,s}$, over the full particle size distribution:

$$R'_c = \int_{d_p^*}^{\infty} 2\pi D_1 d_p (n_1 - n_s S^{d_p^*/d_p}) F(Kn) n(d_p) dd_p \quad (5)$$

The integral can be evaluated by approximating the monomer consumption rate for the polydisperse aerosol by the monomer consumption rate for a monodisperse aerosol. For a monodisperse aerosol, $M'_2/M'_1 = M'_1/N'$. As a result, Eq. 5 can be written as

$$R'_c = 2\pi D_1 n_s (S - S^{d_p^*/d_p}) F(Kn) \frac{M'_2 N'}{M'_1} \quad (6)$$

For $Kn \rightarrow \infty$, and neglecting the Kelvin effect for the stable aerosol, Eq. 6 reduces to the equivalent expression for free molecular growth (Friedlander, 1983) and does not assume that the aerosol is monodisperse.

Zeroth moment

The total particle concentration (zeroth moment) determines the importance of coagulation, which may tend to broaden the particle size distribution. The total particle concentration in the reactor changes along a streamline only as the result of nucleation.

$$\frac{dN'}{dt} = I' \quad (7)$$

First moment

The first and zeroth moments determine the number-average particle diameter $\bar{d}_p = M'_1/N'$. Friedlander (1983) gives the change in the first moment with time as

$$\frac{dM'_1}{dt} = I' d_p^* + \int_{d_p^*}^{\infty} n(d_p) \frac{dd_p}{dt} dd_p \quad (8)$$

The particle growth rate dd_p/dt is given by

$$\frac{dd_p}{dt} = \frac{4D_1 v_1 (n_1 - n_s S^{d_p^*/d_p})}{d_p} F(Kn) \quad (9)$$

and applies to the free molecule, transition, and continuum regimes. If the aerosol growth rate can be approximated by the growth rate of a monodisperse aerosol, the integral in Eq. 8 becomes

$$\begin{aligned} \int_{d_p^*}^{\infty} \frac{4D_1 v_1 (n_1 - n_s S^{d_p^*/d_p}) F(Kn) n(d_p) dd_p}{d_p} \\ = \frac{4D_1 v_1 (n_1 - n_s S^{d_p^*/d_p}) F(Kn) N'}{M'_1/N'} \end{aligned} \quad (10)$$

For $Kn \rightarrow \infty$, and neglecting the Kelvin effect for the stable aerosol, this equation reduces to the corresponding equation for free molecular growth.

Second moment

Following Friedlander (1983), the change in the aerosol surface area with time can be written as

$$\frac{dA'}{dt} = I' k^{*2/3} S_1 + 2\pi \int_{d_p^*}^{\infty} n(d_p) \frac{dd_p}{dt} d_p dd_p \quad (11)$$

If the change in the aerosol surface area with time is the same as that for a monodisperse aerosol, the integral on the righthand side of Eq. 11 can be written, using Eq. 9, as

$$\begin{aligned} \int_{d_p^*}^{\infty} 8\pi D_1 v_1 (n_1 - n_s S^{d_p^*/d_p}) F(Kn) n(d_p) dd_p \\ = 8\pi D_1 v_1 (n_1 - n_s S^{d_p^*/d_p}) F(Kn) N' \end{aligned} \quad (12)$$

Again, for $Kn \rightarrow \infty$ and neglecting the Kelvin effect for the stable aerosol, this equation reduces to the corresponding equation for free molecular growth.

The zeroth, first, and second moments allow calculation of the polydispersity index W , a measure of the spread of the particle size distribution (Pratsinis et al., 1986a). For monodisperse aerosols, $W = 0$; aerosols can be considered approximately monodisperse for $W < 0.3$.

Third moment

A derivation similar to that for the monomer balance results in an expression for the change of the third moment with time.

$$\frac{dM'_3}{dt} = I' k^* d_1^3 + 12D_1 v_1 (n_1 - n_s S^{d_p^*/d_p}) \frac{M'_2 N'}{M'_1} F(Kn) \quad (13)$$

The aerosol yield Y is obtained from the third moment by averaging the point yield y over the reactor volume (Pratsinis et al., 1986a).

Reactor design equations

Equations 1, 7, 8, 11, and 13 can be written in dimensionless form for a tubular flow reactor as

$$2f(r) \frac{\partial S}{\partial \theta} = E \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial S}{\partial r} \right) + R - Ik^* - \frac{4}{3} Kn_1 (S - S^{d_p^*/d_p}) F(Kn) \frac{AN}{M_1} \quad (14)$$

$$2f(r) \frac{\partial N}{\partial \theta} = I \quad (15)$$

$$2f(r) \frac{\partial M_1}{\partial \theta} = Ik^{*1/3} + \frac{4}{9} Kn_1 (S - S^{d_p^*/d_p}) F(Kn) \frac{N^2}{M_1} \quad (16)$$

$$2f(r) \frac{\partial A}{\partial \theta} = Ik^{*2/3} + \frac{8}{9} Kn_1 (S - S^{d_p^*/d_p}) F(Kn) N \quad (17)$$

$$2f(r) \frac{\partial V}{\partial \theta} = Ik^* + \frac{4}{3} Kn_1 (S - S^{d_p^*/d_p}) F(Kn) \frac{AN}{M_1} \quad (18)$$

where $f(r) = (1 - r^2)$ for parabolic velocity profile
 $= 1/2$ for flat velocity profile
 with boundary conditions

$$\begin{aligned} r = 0; \quad \frac{\partial S}{\partial r} = \frac{\partial N}{\partial r} = \frac{\partial M_1}{\partial r} = \frac{\partial A}{\partial r} = \frac{\partial V}{\partial r} = 0 \\ r = 1; \quad S = 1, \quad N = M_1 = A = V = 0 \end{aligned} \quad (19)$$

and initial conditions

$$\theta = 0; \quad S = S_o, \quad N = N_o, \quad M_1 = M_{1,o}, \quad A = A_o, \quad V = V_o \quad (20)$$

For $Kn \rightarrow \infty$, and neglecting the Kelvin effect for the stable aerosol, the equations reduce to the model for Knudsen aerosols given by Pratsinis et al. (1986a) and Kodas et al. (1986). The tubular flow reactor theory with plug flow and without radial monomer diffusion is equivalent to a batch reactor theory.

The dimensionless groups and auxiliary functions are defined in Table 1. Using orthogonal collocation, Eqs. 14–18 were reduced to a set of ordinary differential equations that were solved using the subroutine DGEAR (IMSL, 1980). The aerosol properties in the reactor are determined by six dimensionless groups: Σ , E , R , θ , r , and Kn_1 .

Range of validity of theory

The dimensionless rate of monomer formation can be written as

$$R = \frac{\tau}{\tau_s} \quad (21)$$

where $\tau_s = n_s/R'$ is the time to reach saturation conditions if no

Table 1. Dimensionless Variables, Parameters, and Auxiliary Functions

Group	Physical Meaning
$S = \frac{n_1}{n_s}$	Saturation ratio
$\Sigma = \sigma v_1^{2/3}/k_B T$	Surface tension group
$R = R'/(n_s/\tau)$	Monomer formation rate group
$I = I'/(n_s/\tau)$	
$= \left(\frac{2}{9\pi} \right)^{1/3} \Sigma^{1/2} S^{2-k^*/2}$	Nucleation rate
$\theta = (x/\bar{u})/\tau$	Average residence time
$E = \tau/(b^2/D_1)$	Monomer diffusion group
$A = A'/n_s S_1$	Aerosol surface area
$M_1 = M'_1/n_s d_1$	First aerosol moment
$N = N'/n_s$	Aerosol number density
$r = r'/b$	Radial distance
$k^* = \frac{\pi}{6} \left(\frac{4\Sigma}{\ln S} \right)^3$	Number of monomers in critical size particle
$\tau = \left[n_s \tau_1 \left(\frac{k_B T}{2\pi m_1} \right)^{1/2} \right]^{-1}$	Time for monomer-monomer collisions at saturated conditions, s
$\bar{d}_p = \frac{M'_1}{N'}$	Number average particle diameter
$d_p^* = \frac{4\Sigma v_1^{1/3}}{\ln S}$	Diameter of critical size particle
$Kn_1 = \frac{6D_1}{\bar{c}_1 d_1}$	Molecular Knudsen number
$\bar{c}_1 = \left(\frac{8k_B T}{\pi m_1} \right)^{1/2}$	Monomer mean velocity

monomer losses occur (no radial monomer diffusion) and τ is the time between monomer-monomer collisions at saturated conditions. The time constant for the approach to steady state nucleation, τ_n , can be approximated as (Warren and Seinfeld, 1984)

$$\tau_n \approx \frac{\tau}{S} \quad (22)$$

Thus, Eq. 21 indicates that $R \ll S$ to achieve a steady state subcritical cluster size distribution. For $R \gg S$, particle formation and growth must be described using a dynamic model such as that of Sutugin et al. (1984), which solves for the time evolution of the sub- and supercritical cluster size distributions.

The diffusion group for the tubular flow reactor, E , can be written, using Eq. 22, as

$$E = \frac{\tau D_1}{b^2} = \frac{\tau_n S}{\tau_D} \quad (23)$$

where $\tau_D = b^2/D_1$. This group can be interpreted as the ratio of

the time between collisions of saturated monomer to the time for monomer to diffuse radially from the center of the reactor to the wall. Classical nucleation theory requires $E \ll S$ to reach a steady state cluster size distribution. Thus, for $E \gg S$ particle formation and growth in a tubular flow reactor must also be described using a model that accounts for the time evolution of the subcritical cluster size distribution.

The surface tension group and saturation ratio determine the importance of the Kelvin effect. Calculations indicated that the Kelvin effect did not influence the aerosol dynamics in unseeded tubular flow reactors for most operating conditions. In general, the Kelvin effect is important for small particle diameters and low saturation ratios. During particle formation, however, the saturation ratio is relatively high and the Kelvin effect is unimportant. After particle formation has ceased, average particle diameters are large and the Kelvin effect is again unimportant.

Criteria for production of monodisperse aerosol

The particle growth rate, Eq. 9, can be integrated to give an expression relating the diameter of a particle to its residence time in the reactor and the conditions it is exposed to while in the reactor.

$$d_p = d_{p0} + \int_0^t \frac{4D_1 v_1 (n_1 - n_2 S^{d_p^2/d_r}) F(Kn)}{d_p} dt \quad (24)$$

For Knudsen particles large enough for the Kelvin effect to be unimportant, the integral in Eq. 24 does not depend on particle diameter. This result can be exploited for monodisperse aerosol production. Monodisperse aerosol can be produced by forming small particles by homogeneous nucleation or by seeding the gas, and then growing the particles under uniform conditions. Thus, three conditions must be met to produce monodisperse particles in tubular flow reactors:

1. Particle formation and growth must be separated to insure that initial differences in particle size become negligible relative to the integral in Eq. 24 as the particles grow.
 2. The reactor must have a narrow residence time distribution so that all the particles have the same time to grow.
 3. The particles must be exposed to similar monomer concentrations locally during their residence time in the reactor.
- These criteria are explored further in the discussion that follows.

Separating nucleation and growth

Nucleation and growth can be separated by forming particles in the reactor over a short period of time relative to the reactor residence time, thus seeding the reactor with particles of small average size. Condensational growth of small particles under uniform conditions results in a narrow particle size distribution since initial differences in particle size become negligible as the particles grow, Eq. 24.

To separate nucleation and growth, the average residence time must be much greater than the time at which new particle formation begins,

$$\bar{t} \gg \frac{n_{1c}}{R} \quad (25)$$

or in dimensionless form

$$R\theta \gg S_c \quad (26)$$

The critical saturation ratio can be estimated using various nucleation theories and is usually defined as the value of S required for $I' = 1/\text{cm}^3 \cdot \text{s}$. This simple criterion indicates that for a given average residence time and product, nucleation and growth can be separated by operating with sufficiently high R .

Figures 1a, 1b, and 1c show total particle concentrations, number average particle diameters, and polydispersity indexes for the tubular flow reactor as a function of residence time. The conditions correspond to those of Figure 3 of Henry et al. (1983) for NH_4Cl aerosol production ($T = 295.7\text{K}$, $[\text{NH}_3] = [\text{HCl}] = 25 \text{ ppm}$, $\sigma = 80 \text{ dyne/cm}$). For this relatively high rate of monomer formation (near the limit for the validity of classical nucleation theory), particle formation, Figure 1a, was separated from particle growth, Figure 1b, since $R\theta \gg S$. Particle formation ceased near the reactor entrance, thereby seeding the reactor with particles of relatively small size. As the particles grew, initial differences in size became negligible, resulting in nearly monodisperse aerosol, Figure 1c.

Pratsinis et al. (1986a) used Knudsen aerosol theory in analyzing the experiments of Henry et al. (1983) for the conditions of Figure 1. For these conditions, however, the final particle diameter, $0.4 \mu\text{m}$, was in the continuum regime. Figures 1a, 1b, and 1c compare the tubular flow reactor calculations with those of Pratsinis et al. (1986a). The curve labels are as follows:

Laminar I. Laminar flow reactor with continuum regime growth law and Fuchs-Sutugin correction factor

Plug Flow I. Equivalent to batch reactor with continuum regime growth law and Fuchs-Sutugin correction factor

Laminar II. Laminar flow reactor with molecular bombardment growth law

Laminar II. Laminar flow reactor with molecular bombardment growth law (Pratsinis et al. 1986a)

The form of the growth law had little effect on calculated aerosol properties since the asymptotic particle concentration was reached while the average particle diameter was in the Knudsen regime, where both theories were identical. After particle formation ceased, aerosol growth was limited by the (constant) rate of monomer formation. In general, both theories predicted nearly identical aerosol properties except at low rates of monomer formation.

Nucleation and growth can also be separated by seeding the inlet gas streams. Figure 2 shows the effect of seeding on the polydispersity index for the laminar and plug flow tubular reactors and the continuously stirred tank reactor (CSTAR curve) (Pratsinis et al., 1986a) for the conditions of Figure 1. Seeding batch and tubular flow reactors with sufficiently high concentrations reduces the polydispersity index significantly. The laminar flow reactors gave $W > 0$ because of their distribution of residence times. The plug flow reactors without (I) (equivalent to a batch reactor) and with (II) monomer diffusion both predicted $W \approx 0$ because of their narrow residence time distributions. The aerosol from the plug flow (II) reactor was nearly monodisperse because the seed particles consumed the monomer and thereby minimized monomer diffusion among fluid elements.

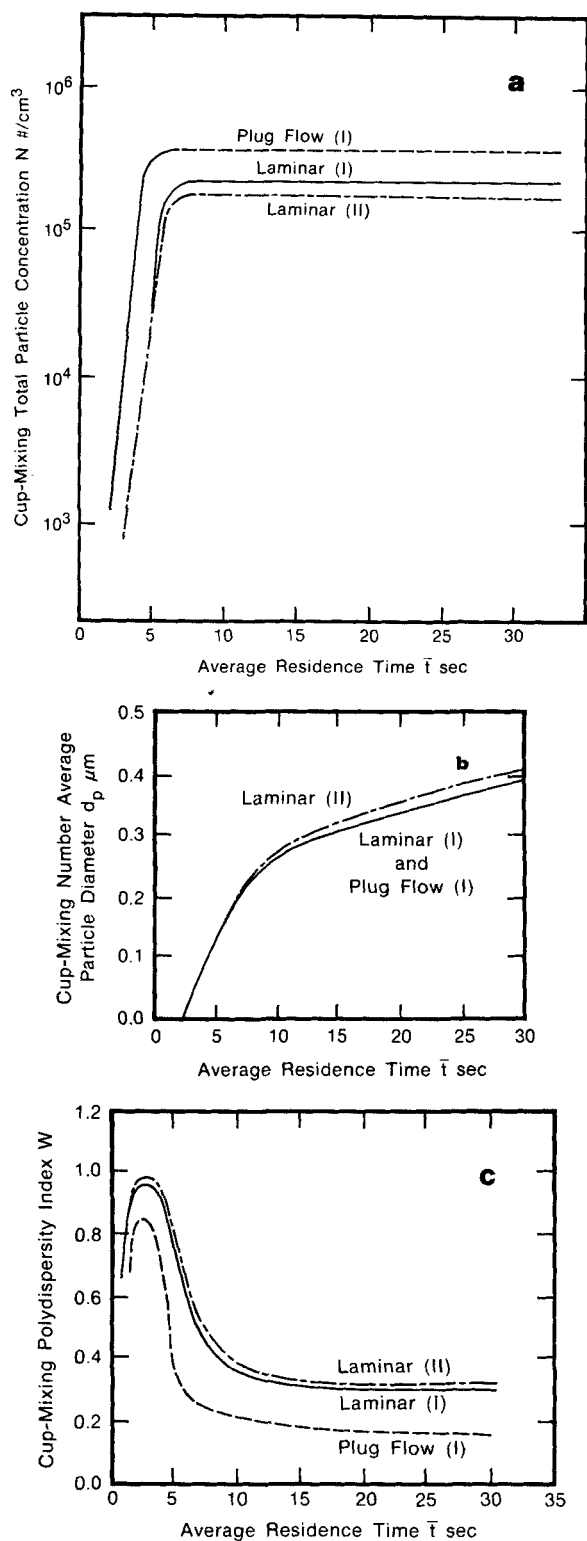


Figure 1. Separation of particle formation and growth.

Curve labels defined in text
 $R = 0.767$; $\Sigma = 2.93$; $E = 4 \times 10^4$; $Kn_1 = 510$; $\theta = 720$

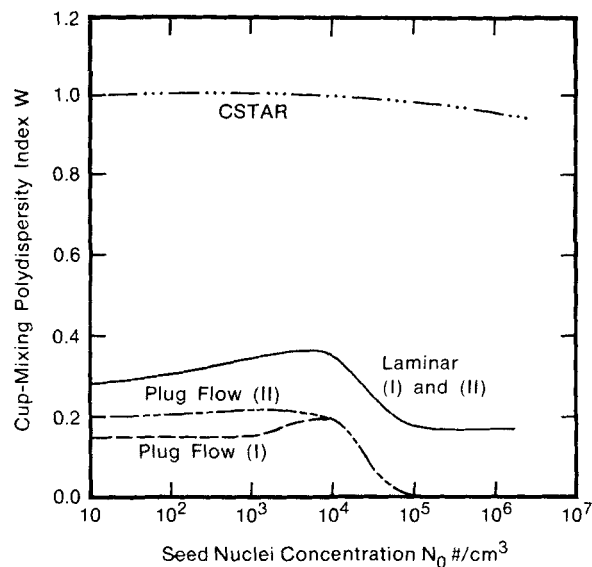


Figure 2. Separation of nucleation and growth by seeding with monodisperse seed aerosol, $d_p = 0.01 \mu$ m, and effect of reactor residence time distribution on spread of particle size distribution.

Results for conditions of Figure 1
 CSTAR, continuously stirred tank reactor
 Plug Flow (II), continuum regime growth law with monomer diffusion
 Other curve labels as for Figure 1

Residence time distribution

Nearly monodisperse aerosol can be produced with reasonably short residence times by operating with a sufficiently high rate of monomer formation, Figure 1. However, a high rate of monomer formation does not insure monodisperse particle production; the reactor residence time distribution must also be sufficiently narrow.

Figure 2 illustrates how the spread of the particle size distribution is determined mainly by the spread in the residence time distribution. The plug flow reactors with the narrowest residence time distributions produced the narrowest particle size distributions for all conditions, while the CSTAR with the broadest (exponential) residence time distribution produced the broadest particle size distribution for all conditions.

Broad residence time distributions in tubular flow reactors have been shown by several investigators to result in polydisperse aerosol formation. Kodas et al. (1987) studied the production of high concentrations of ammonium nitrate aerosol by mixing ammonia and nitric acid in a tubular flow reactor. For high reactant concentrations, mixing effects produced a broad residence time distribution, resulting in polydisperse aerosol with $W > 1$. Raes (1985) studied photochemical sulfuric acid/water aerosol formation in a tubular flow reactor. The measured residence time distribution was exponential, resulting in polydisperse aerosol formation.

Control of radial monomer diffusion

Figures 1 and 2 show that by separating nucleation and growth, tubular flow reactors with narrow residence time distributions can produce nearly monodisperse particles. The particle

size distribution can be narrowed further by minimizing monomer diffusion among fluid elements so that all particles are exposed to the same local concentration of monomer during growth.

Radial diffusion of monomer is negligible when the time to reach the critical saturation ratio is much less than the characteristic time for monomer diffusion to the walls,

$$\frac{n_{1,c}}{R'} \ll \frac{b^2}{D_1} \quad (27)$$

or in dimensionless form

$$\frac{R}{E} \gg S_c \quad (28)$$

For these conditions, a large aerosol surface area is formed near the reactor entrance and consumes the monomer, thereby minimizing radial diffusion of monomer. Thus, for a given condensing species and temperature (which specify S_c), monomer diffusion can be eliminated by operating with sufficiently large R/E . This ratio can be increased by increasing the reaction rate, reactor diameter, or operating pressure. Pratsinis et al. (1986a) have shown that increasing R decreases W .

Figure 3 shows the influence of the monomer diffusion group E on the polydispersity index for the conditions of Figure 1 (Pratsinis et al., 1986b). Reducing E from 4×10^{-4} , Figure 1, to roughly 10^{-6} reduced W from 0.3 to 0.25; monomer diffusion was reduced, thereby narrowing product particle size distributions. For low values of E , the spread in the particle size distribution is determined mainly by the residence time distribution. Increasing E sufficiently also leads to a reduction in W . These conditions are not of practical importance, however, since the number of particles produced is small; most of the monomer is deposited on the walls.

Although plug flow theories (Plug Flow I is equivalent to a batch reactor) can accurately predict the total particle concentration and average size for some cases, Figures 1a and 1b, such theories underpredict the spread in the particle size distribution

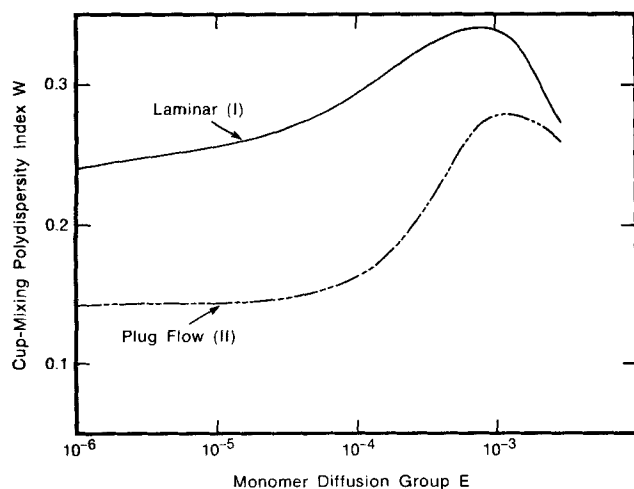


Figure 3. Effect of monomer diffusion group E on polydispersity index.

Laminar and plug flow tubular reactors, conditions of Figure 1

because they neglect the spread in the residence time distribution, Figures 1c, 2, and 3.

Summary

The design of a flow reactor for monodisperse particle production is likely to begin with specification of a total mass production rate and particle diameter for a given material. The parameters controlling aerosol production are Σ , θ , E , and R . For a given material, Σ is fixed by the temperature; tube dimensions, flow rate, and the pressure determine θ and E .

For given Σ , θ , and E , monodisperse aerosol can be produced by operating with a sufficiently high rate of monomer formation, R , the most important control parameter. The upper bound for R is determined by coagulation. The product particle diameter is calculated from R , θ , E , and Σ through the reactor design equations. This is an iterative procedure since the particle diameter calculated at the end will not, in general, be the one originally specified. The number of tubes required is obtained from the total mass production rate and the throughput per tube.

Finally, we wish to note that uncertainties in the nucleation rate require that calculated parameters be tested in laboratory experiments.

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Notation

- A = aerosol surface area density
- b = radius of reactor, cm
- \bar{c}_1 = monomer mean kinetic velocity, cm/s
- d_p = particle diameter, cm
- d_p^* = critical particle diameter, cm
- d_1 = monomer diameter, cm
- D_1 = molecular diffusivity, cm^2/s
- E = diffusion group
- $f = 1 - r^2$ for parabolic velocity profile, $1/2$ for uniform velocity profile
- F = Fuchs-Sutugin correction factor
- I = nucleation rate
- Kn = particle Knudsen number
- Kn_1 = molecular Knudsen number
- k^* = number of monomers in critical size particle
- k_B = Boltzmann's constant
- l = gas mean free path, cm
- m_1 = monomer mass, g
- M_1 = first aerosol moment
- M_2 = second aerosol moment
- M_3 = third aerosol moment
- $n(d_p)$ = particle size distribution function, $\text{no.}/\text{cm}^4$
- n_1 = monomer concentration, $\text{no.}/\text{cm}^3$
- N = aerosol number density
- r = radial distance
- R = rate of monomer formation
- R'_c = rate of monomer consumption by condensational growth, $\text{no.}/\text{cm}^3 \cdot \text{s}$
- $R'_{c,s}$ = rate of monomer consumption by condensational growth of single particle, $\text{no.}/\text{s}$
- S = saturation ratio
- s_1 = monomer surface area, cm^2
- t = residence time, s
- T = temperature, K
- u = fluid velocity, cm/s
- V = aerosol volume density
- v_1 = monomer volume, cm^3
- W = polydispersity index

x = axial coordinate, cm
 y = point yield
 Y = aerosol yield

Greek letters

θ = average residence time
 σ = aerosol surface tension, dyne/cm
 Σ = surface tension group
 τ = time for monomer-monomer collisions at saturated conditions, s
 τ_n = time constant for approach to steady state cluster size distribution, s
 τ_D = characteristic time for radial diffusion of monomer, s
 τ_S = time to produce saturation conditions for monomer, s

Superscripts

' = dimensional
 $-$ = average

Subscripts

o = initial condition
 s = saturation conditions, seed particle
 l = monomer
 c = critical

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